Practice Final Exam
Math 214
(the actual final will be a little shorter)

1.(20 pts) Test the series for convergence or divergence

a) $\displaystyle\sum_{n=1}^{\infty}\left(\frac{3n^2+n+2}{2n^2+4n+7}\right)^n$ b) $\displaystyle\sum_{n=1}^{\infty}\frac{e^n(n+1)^2}{n!}$ c) $\displaystyle\sum_{n=1}^{\infty}\frac{\sin^2 n}{n^2+1}$ d) $\displaystyle\sum_{n=1}^{\infty}\left(1+\frac{1}{n}\right)^n$

2.(10 pts) Determine whether the series is absolutely convergent, conditionally convergent, or divergent.

$$\sum_{n=2}^{\infty}(-1)^{n-1}\frac{1}{n\ln n}.$$

3.(10 pts) Find the radius of convergence and interval of convergence of the series $\displaystyle\sum_{n=1}^{\infty}\frac{(x+1)^n}{\sqrt[3]{n}\;3^n}$.

4.(10 pts) Find the Taylor series for the function $f(x)=x^{-2}$ at $x=1$.

5.(20 pts) a) Find the slope of the tangent line to the curve $r=1+\cos\theta$ at the point where $\theta=\pi/2$.

b) Find the area of the region that lies inside the curve $r=1+\cos\theta$ and outside the curve $r=1$.

6.(10 pts) Find the length of the curve $r=\cos^3(\theta/3)$, $0\le\theta\le\pi/4$.

7.(30 pts) a) Find the area of the triangle with vertices $P_1(2,-1,3)$, $P_2(4,0,3)$, and $P_3(3,-2,4)$.

b) Find an equation of the plane passing through these points.

c) Find parametric equations of the line passing through P_1 and perpendicular to the plane in part (b).

8.(10 pts) Find the vector projection of $\mathbf{b}=\langle 6,2,-4\rangle$ onto $\mathbf{a}=\langle 2,-1,-2\rangle$ and the scalar component of $\mathbf{b}$ in the direction of $\mathbf{a}$.

9.(10 pts) Find the angle between the planes $x+y+3=0$ and $x+2y+2z-1=0$.

10.(10 pts) Find the distance from the point $P(2,-1,1)$ to the plane $3x+y-5z+1=0$.

11.(10 pts) Find parametric equations for the line that is tangent to the curve $\mathbf{r}(t)=\ln(1+t)\mathbf{i}+(1+t)\mathbf{j}-\sin t\mathbf{k}$ at $t=0$.

12.(10 pts) Find the length of the curve
$r(t) = (\sin t - t\cos t)\mathbf{i} + (\cos t + t\sin t)\mathbf{j} + t^2\ \mathbf{k}$, $0 \le t \le 1$.

13.(10 pts) Find the curvature of the curve $r(t) = (\sin t - t\cos t)\mathbf{i} + (\cos t + t\sin t)\mathbf{j} + \mathbf{k}$ at the point where $t = 2$.

14.(10 pts) Find the limit or show that the limit does not exist.

$$\lim_{(x,y)\to(0,0)} \frac{xy^2}{x^2+y^2}.$$

15.(10 pts) Find the derivative of the function

$$f(x,y) = x^2 - 2xy + xz + z^2 + 2x - y$$

at $P_0(1,1,1)$ in the direction of $\mathbf{v} = 2\mathbf{i} - 2\mathbf{j} + \mathbf{k}$. In what direction is the derivative of f at P_0 maximal? Find the derivative in this direction.

16.(10 pts) Find an equation of the tangent plane to the surface $x^2+3y^2+2z^2 = 12$ at the point $P_0(1,1,2)$.

17.(10 pts) Find all the local maxima, local minima, and saddle points of the function

$$f(x,y) = 4x^2 - x^3 + y^2 + 2xy.$$

18.(10 pts) Find the absolute maximum and minimum values of the function $f(x,y) = x^2 + xy + y^2 - 6x$ on the rectangular region $0 \le x \le 5$, $-3 \le y \le 3$.

19.(10 pts) Find the maximum and minimum values of $f(x,y) = x^2 + y^2$ subject to the constraint $x^2 + xy + y^2 = 1$.

20.(10 pts) (**BONUS**) A function $f(x,y)$ is *homogeneous of degree* n (n a non-negative integer) if $f(tx,ty) = t^n f(x,y)$ for all t, x, and y. For such a function (sufficiently differentiable), prove that

$$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = nf(x,y).$$